Astro2020 Science White Paper

Galileo Galilei versus Aristotle, Philosophy of motion, Galileo’s French and English contemporaries, very large numbers, Fermat’s Last Theorem, and speed of light.

Thematic Areas:☐Planetary Systems☐ Star and Planet Formation

☐Formation and Evolution of Compact Objects √ **Cosmology and Fundamental Physics**

☐Stars and Stellar Evolution ☐Resolved Stellar Populations and their Environments

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Abstract(optional): Galileo’s opposition to Aristotle’s concepts of motion developed into square dependence in the philosophical thought on kinematics and dynamics with further introduction of Galileo’s transformation. The interconnection of European philosophers and mathematicians of the time through correspondence of Marin Mersenne and traveling visits developed into very close understanding on the most important concepts including infinite, theory of motion, nature of light and sound, and introduction of analytical geometry that led to the intensive work in the measurement of speeds of light and sound and in the theory of numbers with focus on very large numbers. These concepts were the key factors in understanding the interconnection of time, numbers, finiteness, and infinite in the philosophical thoughts of prominent philosophical figures of the time and were used in their search for solutions or counterexamples to Fermat’s Last Theorem and other Diophantine equations.

"Mathematics is the key and door to the sciences." Galileo Galilei

"Measure what can be measured and make measurable what cannot be measured." Galileo Galilei

"And, believe me, if I were again beginning my studies, I should follow the advice of Plato and start with mathematics." Galileo Galilei.

It is very hard to figure out after many centuries what was the point of dispute between Galileo Galilei and inquisition that included influence of more than two Popes and 20 Cardinals.Some historians, the most notable from almost two dozen names are possibly Jules Speller, Peter Lang, Drake, Blackwell, Pietro Redondi, supported by John Lennox, Wallace, Ferrone, Firpo, and Michael White emphasize that Galileo's confrontation with the inquisition was based on his contradictionof Aristotelian philosophy, based their opinion on direct or indirect examination of different arguments and documents ranging in time from 10 to 20 years before the last Galileo's trial, so let start from few paradoxes:

"Hence time is not number in the sense in which there is 'number' of the same point because it isbeginning and end…The smallest number, in the strict sense of the word 'number', is two…Hence it is so with time. In respect of number the minimum is one so with time. In respect of number the minimum is one (or two); in point of extent there is no minimum....Time is a measure of motion…"Aristotle Physical Treatises Book II, C. Page 300 Chapters 11- 12 220a – 222a

"Salviati: …for we cannot speak of infinite quantities as being the one greater or less than or equal to another. To prove this
Salviati: Very well; and you also know that just as the products are called squares so the factors are called sides or roots; while on the other hand those numbers which do not consist of two equal factors are not squares. Therefore if I assert that all numbers, including both squares and non-squares, are more than the squares alone, I shall speak the truth, shall I not?
Simplicio: Most certainly.
Salviati: If I should ask further how many squares there are one might reply truly that there are as many as the corresponding number of roots, since every square has its own root and every root its own square, while no square has more than one root and no root more than one square.
Simplicio: Precisely so.
Salviati: But if I inquire how many roots there are, it cannot be denied that there are as many as the numbers because every number is the root of some square. This being granted, we must say that there are as many squares as there are numbers because they are just as numerous as their roots, and all the numbers are roots."Galileo "Dialogue on Two New Sciences"

During of the editing of this article, the author found that it is called Galileo's Paradox in numerous books, articles, and many other sources.

And so it could be concluded that Galileo is arguing that the number of squares is equal to the number of all natural numbers, because of infinity argument. Some years ago, knowing only this last conclusion, as it was taught in school, and also forgetting that it is presented in the dialog, the author was much puzzled, on how could this be interpreted.

Also besides, knowing that sequence $1 = 1^2$, $1+3 = 2^2$, $1+3+5 = 3^2$, and $\sum_1^n(2n-1)=n^2$ would generate all the square numbers, and the 1 to 1correspondence of $(2n-1 \leftrightarrow n^2)$ of odd numbers into square numbers and back can be made $\sum_1^n 2n-1 = f(2n-1) = n^2 \Leftrightarrow (2n-1 \leftrightarrow n^2)$, this correspondence look like as shown on the Figure 1:

1 3 5 7 9 11 13 15 17 19. .

1 4 9 16 25 36 49 64 81 100.

Certainly this would apply to the limit also, because it can be simply deduced that

$\lim_{n\to\infty} \frac{n^2}{\sum_1^n 2n-1} = 1 \Leftrightarrow \lim_{n\to\infty} \frac{n^2}{f(2n-1)} = 1$, and it can be deduced that the number of squares is equal to the number of odd numbers. This suggests an explanation of Aristotle in Physical Treatises Book II, Page 280 Chapter 4 202[b] – 203[a], though Aristotle did not mention squares: “Further, the Pythagoreans identify the infinity with the even. For this, they say, when it is cut off and shut in by the odd, provides things with the element of infinity. An indication of this is what happens with numbers. If the gnomons are placed round the one, and without the one, in the one construction the figure that results is always different, in the other it is always the same. But Plato has two infinities, the Great and the Small.”

After figuring this the author found in different books presenting readers expression for squares through odd numbers in connection with Galileo’s work or Ancient Greek Philosophy, and actually this is a simple consequence of the very well known formula from the school:

$\{\sum_1^n n = n(n+1)/2\} \Leftrightarrow \{2\sum_1^n n = n(n+1)\} \Leftrightarrow \{\sum_1^n 2n = n^2 + \sum_1^n 1\} \Leftrightarrow \{\sum_1^n(2n-1) = n^2\}$

This is how it was presented in Hogben 4th Edition Online published 17 years after the 3d. However, in my 3d Edition, I could not find the above relation, rather on page 206 there is relationship between the triangular numbers and squares:1 3 6 10 15 21

. 1 3 6 10 15 21 28

. 1 4 9 16 25 36 49 Figure 2.

This is an equivalent to the sum of 2 consecutive triangular numbers $T_n=\sum_1^n n$, $T_n+ T_{n-1} =\sum_1^n n+ \sum_1^n(n-1) = n^2 \Leftrightarrow \{\sum_1^n 2n-1 = n^2\} \Longrightarrow \lim_{n\to\infty} \frac{n^2}{\sum_1^n 2n-1} = 1$;

It is very difficult to suppose that Galileo did not know these simple equations, as it can be understood from different historians mentioned in Johnson, e.g. professor of Sorbonne Alexander Koyre that suspected that though Galileo mentions experiment with smooth wooden board in sloping position and with the measure of time not exceeding one-tenth of a pulse-beat, there possibly was no experiment, but imaginary demonstration of what Galileo “had figured out mathematically”. Stillman Drake, refuted this theory by examining unpublished pages from Galileo’s notebook. He found notes relating odd numbers to the sequence of squares, Figure 3:

“1 2 3 4 5 6 7 8 ticks (time)

33 130 298 526 824 1,192 1,620 2,104 units of accumulated distance”.

With the correction to 2,104 of the previously written 2,123.

Further the ratios were very close to squares: 130/33 = 3.9 …9.0, 15.9, 25.0, 36.1, 49.1, 63.8.

Galileo recorded how the odd numbers were coming out of ratios of distances:

“130-33	298-130	526-298	824-526.	
97	168	228	298	distances in interval.
97/33	168/33	228/33	298/33.	
2.9	5.1	6.9	9.0	ratio of distances”. Figure 4.

Galileo in the above notes and other works advanced experiment and observation as scientific method to prove mathematical description of a phenomenon, and though familiar with the work of Oresme almost 3 centuries earlier, as seen from Galileo’s diagrams of velocities in the Two New Sciences resembling the triangle graph of Oresme, Galileo is credited with invention of the dynamics as a mathematical science for his doctrines of independence of mass and time in dynamics with his famous Tower of Pisa experiment with falling balls of different masses versus the doctrine of Aristotle “If one weight is twice another, it will take half as long over a given movement.” Book 1, On the Heavens, and in kinematics with the doctrine of distance proportional to the square of time interval for motion with constant acceleration starting from rest.

The other approach is mentioned in the Chapters II and III of Khinchin A.Y.”Three Pearls of Number Theory”. Following new results of Linnik, based on works of Hardy, Llittlewood, Vinogradov, Hua Luogeng, and Schnirelmann’s definition of the density of set S, Khinchin presents interpretation of Lagrange's Four Square Theorem, or Bachet's Conjecture that every positive integer can be written as the sum of at most four squares. The interpretation of this statement is that $\mathbb{N}= A + A + A + A$ where A is the set of all nonnegative squares. Bachet's Conjecture that was stated in the notes of his 1621th translation of Diophantus' Arithmetica is implied from given examples suggesting Diophantus′ knowledge of the theorem. in 1638 Pierre de Fermat, who possessed the very copy of this translation stated without proof his Polygonal Number theorem that generalizes Lagrange’s and triangular cases with statement that every positive integer is a sum of at most *n n*-gonal numbers . However, he presented a proof by infinite descent of Four Squares Theorem, mentioning Bachet and Descartes along with his Last Theorem for the case of n=3 in undated letter. Joseph Louis Lagrange proved the square case only 140 years later. Gauss proved the triangular case 26 years after, commemorating the occasion by writing in his diary the line "EYPHKA!num = Δ + Δ + Δ", and published a proof in his book “Disquisitiones Arithmeticae.” For this reason, Gauss' result is sometimes known as the Eureka theorem. It would follow from the above discussion that if four sequences of 0^2, 1^2, 2^2, 3^2,….k^2,…., would be added together the resulting sequence would contain all natural numbers. The difference from the sum of odd numbers is in that that in the sums $\sum_1^n (2n-1)$of odd numbers the beginning numbers are repeated always, whereas in 0^2, 1^2, 2^2, 3^2,….k^2,…., the beginning numbers would repeat more rarely.

It may be emphasized that authors of the late 30-es and 40-es stressed relationship of triangular numbers and squares, to emphasize concept of density, and 20 years later the authors highlighted connection between odd numbers and squares. Now from the relationship between odd, triangular, and square numbers $T_n + T_{n-1} = \sum_1^n n + \sum_1^n (n-1) = \sum_1^n 2n-1 = n^2$, the paradox in the discussion is well understood, so someone reading the arguments that are written as dispute could be confused to identify the supposed sides arguing pro and con, especially knowing that Galileo prepared to write a treatise on infinite in mathematics. Center of Paris cycle Mersenne believed that "all that is produced is finite but God's potentia is without measure. No created object is adequate to it." (QG: col. 435). Mersenne even expressed concern with Giordano Bruno's ideas not because of atomistic beliefs, but mainly because of his views on the infinity of the World. The year of Beaugrand's visit Cavalieri in a letter to Galileo mentioned his invention about accelerating a wheel to infinity that was quickly refuted by Galileo's argument of obtaining "with these wheels what happens in the case of free fall of bodies".

Galileo also contributed to dynamics analysis of projective motion into a uniform horizontal component and a uniformly accelerated vertical component, he was influenced by the coordinates of Oresme that employed x- and y- axes and were closerto the modern analytic geometry than approaches of Apollonius and Descartes. He showed that the pass of a projectile is parabola and even applied parabola by error to curve suspension of a flexible chain (catena).

French connections and communications were very close that benefited prominent figures of England including Thomas Hobbes and John Pell and throughout Europe. Mersenne introduced Galileo into France, offering 2 years before publication to publish his future work on the "Two Chief Systems of the World", 4 years later he published Galileo's Mechanics with original comments of his own, and 4 years later the "The New Thoughts of Galileo". After Descartes published his "Geometry", he communicated about his achievements to Galileo.

Fermat developed analytic geometry closer to the modern version and 8-9 years before Descartes' La géométrie by application of Viète's algebra to Apollonius's analytic geometry approach. Fermat communicated his ideas and findings in letters, but his book "Introduction to Plane and Solid Loci", written possibly around the same time was not published until some years after his death. He obtained subtangents to the ellipse, cycloid, cissoid, conchoid, and quadratrix and areas of parabolas and hyperbolas with most of the curves related to quadratic forms. It is highly probable that the pupil of Viete, Jean Beaugrand, who was a very close friend of Fermat even 2 to 3 years before Fermat's works on analytic geometry and already corresponded works of Fermat to Marin Mersenne's cycle in Paris, notified Galileo and other scholars in Italy about Fermat's achievements, when 6 years later he visited Cavaliery in Bologna, Castelli in Rome, and Galileo, who was confined to house arrest in Arcetri.

The year Fermat began work on analytic geometry, even before Galileo's publication of his research on the motion, Cornier, Descartes, and Beeckman tried in letters to persuade Mersenne, who later convinced himself that in the motion of free fall "the spaces grow in the duplicate ratio of the times." Mersenne thought that Aristotle and Kepler were wrong in believing that light speed is infinite and thus, the idea of measuring speed of light was following next. Mersenne began working on optics and acoustics with publication of different Treatises right after Bachet's translation of Diophantus. Mersenne is credited with measuring speed of sound $1/10^{th}$ close to the correct one, two years after Galileo's conclusion in his famous experiment that light travels at least 10 times faster than sound that credited him with being the first to determine the speed of light. His simple method shows his astonishing biosensual abilities to determine short time intervals, mentioned in wooden board experiment even losing vision by that time. The actual ratio of speed of light to speed of sound is 874030. Less than 40 years later Ole Romer applied Galileo's method of establishing time of day and longitude by making observations of Jupiter's moon Io discovered by Galileo to determine the speed of light $1/4^{th}$ to $1/5^{th}$ close to correct one.

Two years before Galileo's experiment Mersenne expressed his beliefs on the 2,000 years old discussion on the nature of light in L'Harmonie Universelle using newly introduced term “diaphanous ” by Sir Walter Raleigh with roots from Greek and Latin in his History of the World written, while he was imprisoned in the Tower of London. Mersenne declared that “light is the soul of the air, and of the other diaphanous bodies”, and that “it can be named the universal soul of the world, which is somehow similar to the death, when it is prived from light.” It is possible that he associated time with numbers and considering that all the prominent philosophers and mathematicians of the time were concerned with integer solutions to Diophantine equations, quadratic forms, questions of finiteness and infinity, it is very probable that Galileo’s speed of light experiment was in support of possibly suspected some kind of Einstein’s transformation, especially after well known Theory of Galileo’s transformation described in his “Dialogue Concerning the Two Chief World Systems” His claim that he did not perform his thought experiment credited Pierre Gassendi 2 years after Galileo’s experiment on speed of light with being first, who performed it in the presence of Louis de Valois, Governor of the Provence in Marseilles with dropped ball on resting or fast moving ship. His description was written along with requested by Mersenne answers to Descartes in his “Impressed by the Mover of the Motion...”

“Mathematicus retained in the sixteenth and seventeenth centuries the meaning it had for the Middle Ages; it meant; ‘astrologer’ or ‘astronomer’” (Wagner citing Mahoney). Thus, it is possible that Galileo’s experiment contributed to his search of anticipated solutions to $x^n + y^n = z^n$ (Fermat’s Last Theorem, FLT) stated according to some sources 8-9 years before, and for n=3 first stated by astronomer Al-Khojandi from near Iran 650 years before, possibly known from the time of transfer of Algebra from Middle East to Europe. Around 150 years after Fermat the Head of Gottingen Observatory Gauss in the letter to astronomer Wilhelm Olbers [Gauss Werke, Vol. 2,629] called FLT one “of such propositions, which one could neither prove nor disprove”. Fermat’s letters to Mersenne and to Frenicle de Bessy, where he stated cases of n=3 and 4 are dated the same years of Galileo’s and Mersenne’s experiments on speed of light and sound. Wagner considered that problem in Fermat’s letter to Mersenne dated three years later was related to solution of FLT: “To find a pythagorean triangle in which the hypotenuse and the sum of the arms are squares” provided with Fermat’s answer “(4565486027761, 1061652293520, 4687298610289)”. Bessy gave own proof for case n=4 of FLT, challenged Christian Huygens to solve similar problem on the system of equations in integers: $x^2 + y^2 = z^2$, $x^2 = u^2 + v^2$, $x - y = u - v$, solved by Theophile Pepin 240 years later, and had knowledge about number $1729=9^3 + 10^3= 12^3 + 1$. Sequence of near misses by 1 in FLT for n=3: $6^3 + 8^3 = 9^3 - 1$, $135^3 + 138^3 = 172^3 - 1$, $791^3 + 812^3 = 1010^3 - 1$, $11161^3 + 11468^3 = 14258^3 + 1$, $65601^3 + 67402^3 = 83802^3 + 1$, can be continued with very large numbers, e.g. $459818495907977390827136308815215025^3+554871986162401780953595961082843650723^3 =644775824628777984478334962509496242333^3-1$ that likely was understood by Bessy.

It can be concluded that sharp turn to square dependence in the philosophical thought in the kinematical and dynamical approaches to the theory of motion, Galileo’s transformation, the interest in very large numbers, measurement of speeds of light and sound, astronomical tables of Kepler, astronomical discoveries, e.g. Jupiter’s moons by Galileo and other, along with new astronomical methods for finding longitudes, with deep belief of time and numbers connection, all of these factors not only were connected in the philosophical thoughts of Galileo, Mersenne, Fermat, Descartes, and other prominent philosophical figures of the described time, but also were used in their search for solutions or counterexamples to Fermat’s Last Theorem and other Diophantine equations.

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