Astro2020 Science White Paper

Application of timescale, magnitudes, and fractal time with Stein estimators to astronomical observations.

Thematic Areas: Planetary Systems Star and Planet Formation

☐Formation and Evolution of Compact Objects Cosmology and Fundamental Physics

☐Stars and Stellar Evolution Resolved Stellar Populations and their Environments

Galaxy Evolution Multi-Messenger Astronomy and Astrophysics

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Abstract: Discussion of definite and sometimes enormous difference in application of spatial time multiscale analysis, multidimensional and multifractal time analysis with different magnitudes through examples of cyclotomic polynomials in Number and Algebraic theories and Doodson coefficients in Tidal Theory analysis with Kolmogorov's 5/3 law in the equations of turbulence applied by Tatarski to astronomical observations and later to Galaxy suggests application of Stein type estimators to more accurate orbits evaluation with effect attributed by Poincare to Kepler and Tycho.

In the Book of Genesis the letter 'Gimel' for the first time is at the 738th place and in the 190th word in the Passage about the Creation of Sun and Moon in the Fourth Day of Creation, where $738–3^2–1^2 = 728 = 27^2 – 1^2 = 9^3– 1^3= 6^3+ 8^3$, and $190 = 10^2+9^2+ 1^2= 11^2+8^2+ 2^2+ 1^2=12^2+6^2+ 3^2+ 1^2= 13^2+4^2+2^2+1^2 =11^2+8^2+ 2^2+ 1^2=10^2+8^2+ 5^2+ 1^2 = 5^3+ 4^3+ 1^3= 6^3-5^2- 1^2$ that could have been known by astronomer Galileo from his Paduan student of seven years Joseph Solomon Delmedigo also an astronomer, who was mentioned as the Paduan Rebbi. This is one of the possible explanations, why Galileo's Dialogues were divided into four dialogues or days. Ten years after Delmedigo's graduation Mersenne published three volumes commentary on the first 6 Chapters of the Book of Genesis that covers almost 1900 folio columns; $1900= 12^3+5^3+6^2$.

The science of the nature is concerned with special magnitudes and motion and time, and each of these at least is necessarily infinite or finite, even if some things dealt with by the science are not, e.g. a quality or a point --- it is not necessary that such things should be put under either head. (Aristotle Physical Treatises Book II, C. Chapter 4 202^b – 203^a Page 280).

Kepler remarks that the positions of a planet observed by Tycho are all on the same ellipse. Not for one moment does he think that, by a singular freak of chance, Tycho had never looked at the heavens except at the very moment when the path of the planet happened to cut that ellipse. What does it matter then if the simplicity be real or if it hide a complex truth?.... We draw a continuous line as regularly as possible between the points given by observation. Why do we avoid angular points and inflexions that are too sharp? Why do we not make our curve describe the most capricious zigzags? It is because we know beforehand, or think we know, that the law we have to express cannot be so complicated as all that. The mass of Jupiter may be deduced either from the movements of his satellites, or from the perturbations of the major planets, or from those of the minor planets. If we take the mean of the determinations obtained by these three methods, we find three numbers very close together, but not quite identical. This result might be interpreted by supposing that the gravitation constant is not the same in the three cases; the observations would be certainly much better represented. ***Why do we reject this interpretation? Not because it is absurd, but because it is uselessly complicated***." (Henri Poincare Science and Hypothesis Chapter 9: Hypotheses in Physics pp.143-145 149-150)

"Lemma 2 The moment of any genitum is equal to the moment of the generating sides multiplied by the indices of the powers of those sides, and by their coefficients continually…and in general, that the moment of any power $A^{\frac{n}{m}}$ will be $\frac{n}{m}A^{\frac{n-m}{m}}$ "

(Isaac Newton Mathematical Principals Book II The Motion of Bodies Page 108)

Most of the people are concerned with the philosophy of the "motion" or "time", or with "finite" or "infinite", and these philosophies were subjects of more than 2,000 years old controversy, it even follows from Mersenne's opinion that the main reason for prosecution of Giordano Bruno was a question of infinity or finiteness of the World. Other people would probably concentrate on "quality" or a point". But few would pay attention to Aristotle's "special magnitudes", it even became customary not to consider this term simply, because it is obviously related to measurement, though this would not be satisfactory explanation why Aristotle placed "special magnitudes" before the "motion" and "time".

Could study or investigation of “special magnitudes” explain Mersenne’s mysterious view versus infinity of natural numbers: “*all that is produced is finite* but God's *potentia is without measure*. *No created object is adequate to it.” (QG*: *col. 435*)? Previously the author had given an explanation that the statement was possibly related to the search of counterexample or solution to the 650 years old statement of Fermat's Last Theorem, stated with flawed proof for n=3 by Arabic astronomer Al-Khojandi and possible relation of the statement being absolute or not to the measurement of speed of light and sound, subject of more than 2,000 years old controversy and were measured first correspondingly by Galileo and Mersenne in years corresponding to the letters of Fermat, where he inquired about finding such cubes or biquadrates. Paulo Ribenboim in “13 Lectures on Fermat's Last Theorem” pp. 1-2 brought to light that “Tannery (1883) mentions a letter from Fermat to Mersenne (for Sainte-Croix) in which he wishes to find two 2 cubes whose sum is a cube, and two biquadrates whose sum is a biquadrate. This letter appears, with the date June 1638, in volume 7 of Correspondence du Pere Marin Mersenne (1962); see also Itard (1948). The same problem was proposed to Frenicle de Bessy (1640) in a letter to Mersenne, and to Wallis and Brouncker in a letter to Digby, written in 1657, but there is no mention of the remarkable proof he had supposedly found. In modern language, Fermat's statement means: The equation $X^n + Y^n = Z^n$, where n is a natural number larger than 2, has no solution in integers all different from 0.”
Bessy had knowledge about number $1729=9^3 + 10^3= 12^3 + 1$. Sequence of near misses by 1 in FLT for n=3: $6^3 + 8^3 = 9^3 - 1$, $135^3 + 138^3 = 172^3 - 1$, $791^3 + 812^3 = 1010^3 - 1$, $11161^3 + 11468^3 = 14258^3 + 1$, $65601^3 + 67402^3 = 83802^3 + 1$, could continue by big numbers, e.g. $4598184959079773908271363088152150 25^3+5548719861624017809535959610828436 5072^3 ==6447758246287779844783349625094962 4233^3-1$.

The other explanation is that Mersenne contrary to Kepler, Galileo, Laplace, Gauss and other polymaths was not an astronomer, and possible meaning of the word “produced” may be “by man”.
In any event one of the most notable emergences of his statement surfaced in connection with cyclotomic polynomials, where in the theory of Diophantine approximations, and in particular in theory of transcendental numbers, the size of polynomial is expressed by its height (maximum of the magnitudes of its coefficients) or length (the sum of the magnitudes of its coefficients). The vast research on the theory of cyclotomic polynomials focused on the magnitude of the height A(n) or the largest coefficient or on determining the degree of $\Phi n(x)$ in the study of divisors of $x^n\text{-}1=\prod_{d|n} \Phi_d (x)$ with the product over all the divisors d of n with some basic properties Φ_1 (x) = x-1 for d= primes p or q, $x^p - 1 = \Phi\, p(x)\Phi_1(x)$,
$x^{pq} - 1 = \Phi pq(x)\Phi_p(x)\, \Phi q(x)\Phi_1(x)$, $x^6\text{-}1=\prod_{d|6} \Phi_6 (x)\Phi_3 (x)\, \Phi_2 (x)\, \Phi_1 (x)$, for prime d Φ_d (x) $= x^{d-1}+ x^{d-2}+\ldots+x^2+x+1$ with the ***enigma*** arising from the first steps and growing even further $\Phi_8 (x)= \Phi_4 (x^2) = \Phi_2 (x^4)$, but $\Phi_6 (x) \neq \Phi_3 (x^2)$ and $\Phi_6 (x) \neq \Phi_2 (x^3)$. The magnitude, or the height of all these polynomials equals to 1 (the coefficients of Φ_d (x) are 0,1, or -1).

In many mathematical problems that require many multidimensional investigations including the research on the theory of cyclotomic polynomials, the intuition or some mathematical sense directs mathematicians to choose directions of research or anticipate results, as it is the case with numerous Number Theoretical or Group Theoretical Conjectures that wait centuries for a proof or their refutation or counterexample. And it is not a surprise that the same year that Kronecker, who was the worst enemy of Cantor for his set theoretical approach with great philosophical disagreements between them, wrote more than 120 pages long article on

factorization of 2 polynomials, the professional correspondence between Dedekind and Cantor stopped after Dedekind's refusal of Cantor's offer to accept the chair position at Halle. Other mathematicians, listed by Cantor, who refused this position, were Einstein's undergraduate professor Weber and Mertens. Next year Migotti discovered an example of Φ_{105} (x)= Φ_{3*5*7} (x) of a polynomial with degree 48 and a magnitude (height, with 2 coefficients equal to -2) of 2. And the whole Cantor's theory starting in the first proof with evaluation of the height of the polynomials became very much endangered; there seemed to be nothing that could save it. The very same fate could happen to argument of new number generation from given sequence or in diagonal argument, after few steps the sequence in construction can swing in unpredictable move, especially after the ***recent findings*** enormously increasing the mystery surrounding cyclotomic polynomials, due to Gallot and Moree on difference in neighboring coefficients of ternary (n equals product of 3 primes) cyclotomic polynomials by at most one, e.g. in Φ_{105} (x) coefficients are: 1,1, 1, 0, 0,−1,−1,−2,−1,−1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0,−1, 0,−1, 0,−1,0,−1, 0,−1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0,−1,−1,−2,−1,−1, 0, 0, 1, 1, 1. From the next numbers with 3 odd prime factors 165 =3*5*11, 195 = 3*5*13, 210 = 2*3*5*7, 255 = 3*5*17, 273 = 3*7*13, 285 = 3*5*19, Φ_{231} (x)= Φ_{3*7*11} (x) puzzlingly has height magnitude of 1 (all coefficients are 0,1, or -1). G. Bachman showed that there are infinitely many ternary cyclotomic polynomials Φ_{pqr} (x) with height magnitude A(pqr)=1. Largely, though there are some recent results on periodicity and connection between ternary and binary cyclotomic polynomials as a rule the value of coefficients of ternary cyclotomic polynomials is not known, and it took Bang 12 years after Migotti's finding to obtain some bound on their height magnitude that was improved only after 73 years by Beiter. But after Migotti's finding the growth of the height magnitude of coefficients of cyclotomic polynomials with growing number of primes in the product could have been anticipated. However, this growth is so mystifyingly astonishing that it is possible to speculate that it was not expected for around a century, or possibly few decades less, e.g. for the products of 9 odd primes, magnitudes are
A(3*5*7*11*13*17*19*23*29=3234846615) = 2,888,582,082,500,892,851;

A($\prod$ 3,5,7,11,13,19,29,37,43)=5,465,808,676,670,557,863,536,977,958,031,695,430,428,633

Where for n = 3*5*7*11*13*19*29*37*43= 13 (3*5*7=105)($11*19*29\sim$ 6,000)($37*43\sim$1,550)~13 billion=1012520355 *13=13,162,764,615, the degree of Φn(x) is between 4 and 5 billion, however, the product of 10 primes is yet inaccessible for the modern computers. Next year Cantor became mentally ill with illness following him for the rest of his life, and a year later Mittag-Leffler became concerned with philosophical issues and terminology in Cantor's paper and subsequently asked Cantor to withdraw the paper from Acta Mathematica that was founded by Mittag-Leffler 3 years before, when the paper was still in proof with remark that it was "... about one hundred years too soon."
Speculation by Fernando Q. Gouvêa. in the very recent article Was Cantor Surprised? about Cantor's dissatisfaction and disbelief with his own proof is even more critical than presentation by Julius König at the Third International Congress of Mathematicians about falseness of transfinite set theory, philosophical objections of Wittgenstein or criticism by Henri Poincaré, L. E. J. Brouwer, and Hermann Weyl, who was a colleague of Albert Einstein at Zurich of Cantor's theory as counter-intuitive, and concerns of famous mathematicians' E. T. Bell, Geoffrey Hardy, E. M. Wright, G. Birkhoff and S. Mac Lane that followed Perron's opinion about non-constractive property of Cantor's proof.

Following Ptolemy's Tetrabiblos that explaned appearance of the tides by the influence of the Moon derived from ancient observation, Johannes Kepler attributed cause of tides to the gravitation of the Moon supporting his argument by ancient observations and correlations. However, 7 years later Galileo Galilei writing an apology letter, he called “Discourse on the Tides” to Cardinal Orsini related cause of tides to Earth's rotation and revolution around the Sun. 70 years later Sir Isaac Newton, in Principia Mathematica focused his explaination of ocean tides on comparison of Newton’s law of gravitation for gravitational attraction between two bodies being proportional to the product of their masses, and inversely proportional to the square of the distance between the bodies and tidal generating forces that vary inversely as the cube of the distance from the tide generating body. Even the gravitational force between the Earth and the Sun is more than 177 times that between the Earth and the Moon, tides are caused mainly by the Moon. Mass(Sun) $\approx$ 27,000,000 Mass(Moon), but Dist(from Sun) $\approx$ 390 Dist(fromMoon).
Thus, Sun tide-generating force / Moon tide-generating force is $\approx 27{,}000{,}000/390^3 \approx$ 27/59,319 or less than a half. 90 years later Pierre Simon Laplace developed the dynamic theory of tides that accounts for friction, resonance and natural periods of oceans, and effects of the continents with explanation of tides interactions with deep sea ridges and mountain chains of seamounts and rising deep eddies from the deep to the surface. Satellite observations confirm the accuracy of the dynamic theory, and the tides worldwide are now measured to within a few centimeters. Special attention should be given to stunning analogies and parallels between Doodson coefficients for the Lunar diurnal tides and the height magnitude of coefficients of the first cyclotomic polynomials including binary and ternary. Doodson included around a century ago close to 400 components with more than 60 main components with the further enlargement in the number of variables in the modern theory. The components are adjusted to consider the angles of rotational and spinning planes of Earth, Sun, and Moon at different instances of time. The theory also observes spatial structure of tides. Considering that apogee and perigee of the Earth and Moon are contributing to the low and high tides, the theory can adjust for differences in Earth gravitation with the reciprocal of the flattening 1/f of Earth ellipsoid equal to 298.257223563 with slow variations of the bulge at the equator already mentioned by Poincare that had been decreasing, but for the last 20 years has been increasing because of redistribution of ocean mass through currents and the motion of the Earth’s magnetic field (North magnetic Pole). Further considering that approximate ratio of dynamic and equilibrium theories of tides is more than 30 times, all the above suggests the use of time scale and fractal time analysis with application of Stein type estimators in evaluation of tidal time series analysis.
Almost 80 years ago A.N. Kolmogorov applied dimensionality considerations to the equations of turbulence with very large Reynolds number to obtain equations for spatial time scale analysis with actually multidimensional and fractal spatial time based on different assumptions like L.F. Richardson's little poem: “Big whirls have little whirls That heed on their velocity, And little whirls have littler whirls And so on to viscosity.” Also because of the conditions and wave behavior in the problem the results and derivation is highly applicable in astronomical observation characterized by multifractal and multidimensional space and time analyses. 20 years later Kolmogorov applied his results again to intermittency problem in turbulence with very large gradients and velocity differences and is multiscale phenomenon with possible ratio of largest and the smallest lengths is of $10^5 – 10^6$, and waves are composed from

1] unstable large eddies(energy-containing length scale that break up into small ones and
2] the energy is transferred from large scales to smaller until such length scale (dissipation scale η) that the memory friction of the fluid dissipate the kinetic energy. Kolmogorov considered them to be probabilistically and statistically independent. The motion of dissipation scale η is close to statistical equilibrium (`equilibrium range'), and statistics in the equilibrium range are uniquely determined by the memory friction γ and the rate of energy dissipation or energy flow ϵ. The dimension of energy is mass*lenght^2/time^2, ϵ has dimension energy/mass*time= lenght^2/time^3. The wave number (number of waves in the unit of length) k has dimension 1/length, and Energy spectrum E(k) is the energy (for constant mass)transported by waves in the interval between two wave numbers $E(k)\Delta k = E(k)\,(k_1-k_2)$, and its dimension would be energy/(mass*wave number) = lenght^3/time^2. From physical and invariance, and isotropy at different scales and directions considerations $E(k) = C\,\epsilon^x k^y$, where C is dimensionless constant. Solving system of equations for lengths and times lenght^3= lenght^{2x}lenght^{-y}, time^2= time^{3x} , the equation becames $E(k) = C\,\epsilon^{2/3} k^{-5/3}$ is Kolmogorov's 5/3 law, constant C is determined by experiments to be close to 1. Though the assumptions are heuristic and results slightly vary in different experiments, the theory of multifractal spatial time scale in multidimensional problems was further developed in the 60-es and 70-es by Tatarski and after application to astronomical seeing with effects of apparent burring in optical observations (e.g. "Martian canals") was applied in the 80-es to Galaxy observations.
Fermat calculated the subtangent to the ellipse, cycloid, cissoid, conchoid, and quadratrix the area between semicubical parabola $y^3 = p x^2$, the x axis , and x = a to be $3/5p^{1/3}a^{5/3}$ that has similarity with Kolmogorov's 5/3 law.
Fibonacci(n)= $(\varphi^{n} - (-\varphi)^{-n})/\sqrt{5}$ = round $(\varphi^{n})/\sqrt{5}$ = round $(\varphi^{n+1/3})$, or $[(\varphi^{n+1/3})]$ that is correct for few first Fibonacci numbers, thus implying that the Newton's Lemma, mentioned in the very beginning could have been guessed long before. Figure 1 shows Fibonacci numbers relations to approximate distances of the Planets from the Sun in AU. Remembering Poincare's mention of Kepler's remark on planets' position during observations all of the considerations suggest use of Stein type estimators for calculation of their orbits instead of usual least squares estimator used by Gauss for finding Ceres orbit. Stein worked for Air Force during WWII and after introduction of his estimator was elected to National Academy. It is called Stein's Phenomenon or Paradox. For known covariance matrix in n-variate $\boldsymbol{X} \sim N(\boldsymbol{\mu}, \sigma^2 I_n)$ Stein estimator $\boldsymbol{\mu}_s=(1-(n-2)\,\sigma^2/\|\boldsymbol{X}\|^2)\,\boldsymbol{X}$ has less mean squared error than usual maximum likelihood estimator for n>2. That means that for dimensions 3 and higher Stein estimator has greater accuracy. The method is used in weather forecasting and other applications. This method with multidimensional approach can be applied in search of orbits of Dwarf Planets or other Planets and Stars.

Planet	Mean distance from Sun(AU)	F(n)/3
Mercury	0.4	1/3
Venus	0.7	2/3
Earth	1.0	3/3
Mars	1.5	5/3
Asteroids	2.8	8/3
Jupiter	5.0	13/3
Saturn	10	34/3
Uranus	20	55/3
Neptune	30	89/3
Pluto	40	144/3

Figure 1.

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