AFOSR contract FA9550-15-1-0043

"Electromagnetic modeling, optimization and uncertainty quantification for antenna and radar systems, rough surfaces scattering and energy absorption"

Final Report

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1 Introduction

This effort concerns a variety of mathematical problems in the field of electromagnetic propagation and scattering, with applicability to design of antenna and radar systems, energy absorption and scattering by rough-surfaces. This work has lead to significant new methodologies, including introduction of a certain

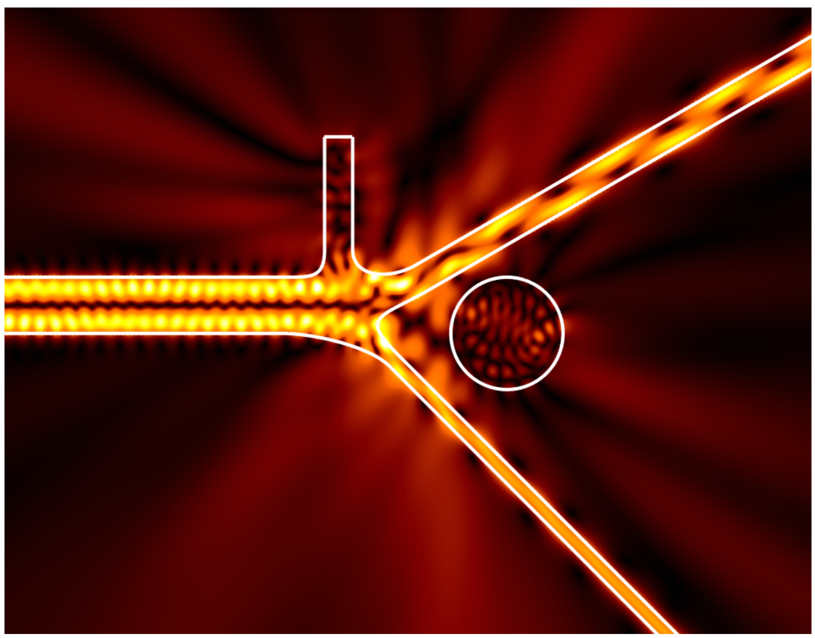


Figure 1: Absolute value of u_w produced by the WGF method for a complex open-waveguide problems.

Windowed Green Function method (WGF), which gives rise to electromagnetic isolation in the solution process and thereby enables effective use of hybridization of scattering solvers (Section 2), it has lead to effective methods for simulation of *Dielectric antennas and multi-material electromagnetic structures* (Section 3), it has resulted in a novel high-order *Rectangular integration method* which, relying on surface descriptions by non-overlapping patches, is well adapted to integral-equation solution on surfaces given in formats derived from Computer Aided Design, also known as CAD (Section 4), and it has lead to new solvers for problems of *Scattering by periodic arrays of cylinders at Wood-anomalies* (Section 5) as well as *Explicit, implicit and explicit-implicit time-domain FC methods of high-order of time accuracy* for general hyperbolic and nonlinear parabolic systems—with application to the Maxwell system, the elastic wave equation, the Navier-Stokes equations, etc (Section 6). Our conclusions are presented in Section 7.

2 The Windowed Green Function method

The WGF method [20] is based on use of certain families of smooth windowing functions supported on regions of varying diameter A, which raise from zero to one and then decay back to zero, and which do this in a *slow* manner—in such a way that all spatial derivatives of the windowing function tend to zero as $A \to \infty$. We have shown that when used in conjunction with appropriate Green's functions, the windowing functions can significantly simplify the treatment of problems posed by hybridization of a variety of numerical solvers. Even the simplest examples of application of this method provide powerful solutions: recent applications to the problems of scattering by an obstacle in presence of layered media and open-waveguide junctions (including launching and termination of dielectric waveguides and antennas) have been highly successful for these conceptually simple but important and famously challenging problems. In particular, the WGF method yields numerical results of excellent quality for open-waveguide and dielectric antenna problems, with highly accurate solution of problems including waveguide junctions as well as waveguide termination and launching/illumination.

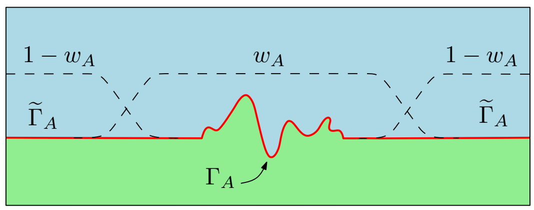


Figure 2: Window function w_A and the windowed sections Γ_A and $\widetilde{\Gamma}_A$ of the unbounded curve Γ.

A well known previously existing method, which relies on use of Sommerfeld-integrals (see [18, 19, 26, 28] and references therein) can be applied to some of these problems. As demonstrated in reference [20], however, even in this case the windowing approach is highly competitive: in some simple examples considered in that reference the windowing-based computing times are several orders of magnitude smaller, for a given accuracy, than the corresponding times required by Sommerfeld-integral methods. For the corresponding problem of junction of optical fibers in three dimensions, in turn, we do not know of any reasonably feasible alternative.

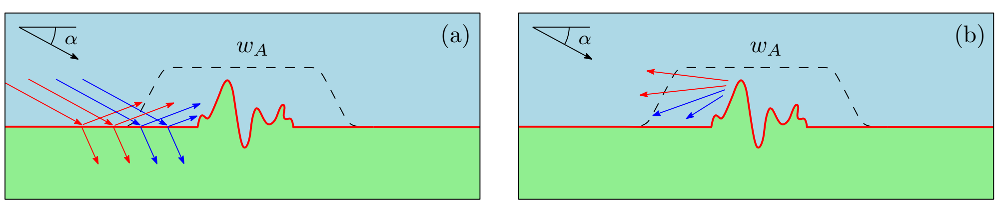


Figure 3: Physical concepts underlying the WGF method.

In all generality, further, when used in conjunction with integral equation formulations of the a Maxwell problem, the slow-rise windowing functions can be made to act as electromagnetic isolators: the resulting windowed integral equations effectively separate the windowed region from the remainder of the structure, and thus yield the current produced solely by direct impact of any given incident field—without edge effects and without any errors other than those that arise from multiple scattering from regions eliminated by the windowing process. The windowed operators effectively isolate a selected region of a scattering structure without introducing spurious reflections, and they therefore provide an excellent basis for an overall multiple-scattering approach. A crude separation of a substructure would result in completely incorrect results, including, most importantly, reflections arising from the edges created in the partitioning process and mathematical difficulties arising from the conversion from a closed surface to an open surface with unphysical highly-reflecting edges. The slow rise windowing function, in contrast, truly isolates a selected portion of the scattering structure from the remainder of the structure *without introduction of unphysical edge effects!*

3 Dielectric antennas, open-waveguide eigensolvers, and multi-material electromagnetic structures

This effort includes numerical simulation of dielectric-antenna systems. A certain " mode-matching" approach, which was mentioned to us by Air Force scientist Dr. Naftali Herscovici, was included amongst other alternatives in the grant proposal that lead to this effort. After investigating various mode-matching strategies we concluded that such methods may not prove useful in view of their lack of sufficient space resolution: for transverse magnetic problems (Neumann boundary conditions) it does not appear generally possible to achieve convergence on the basis of such methodologies.

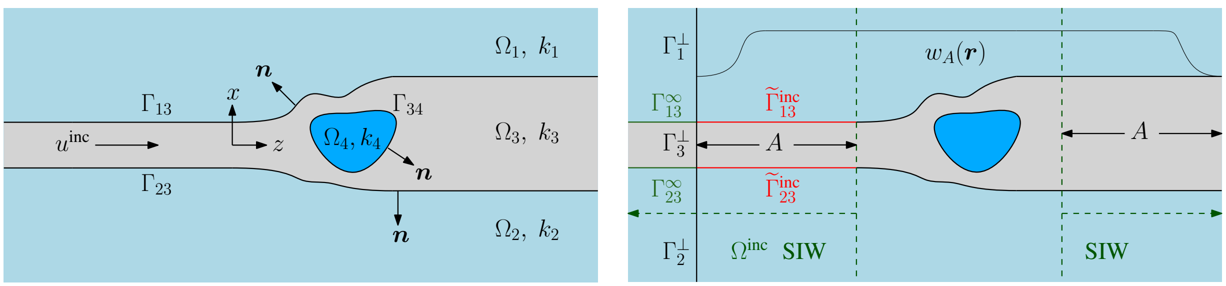


Figure 4: The open-waveguide problem and geometrical structures utilized in the WGF method developed as part of this effort. Left: Schematic depiction of a typical waveguide scattering problem. Right: Waveguide WGF setup.

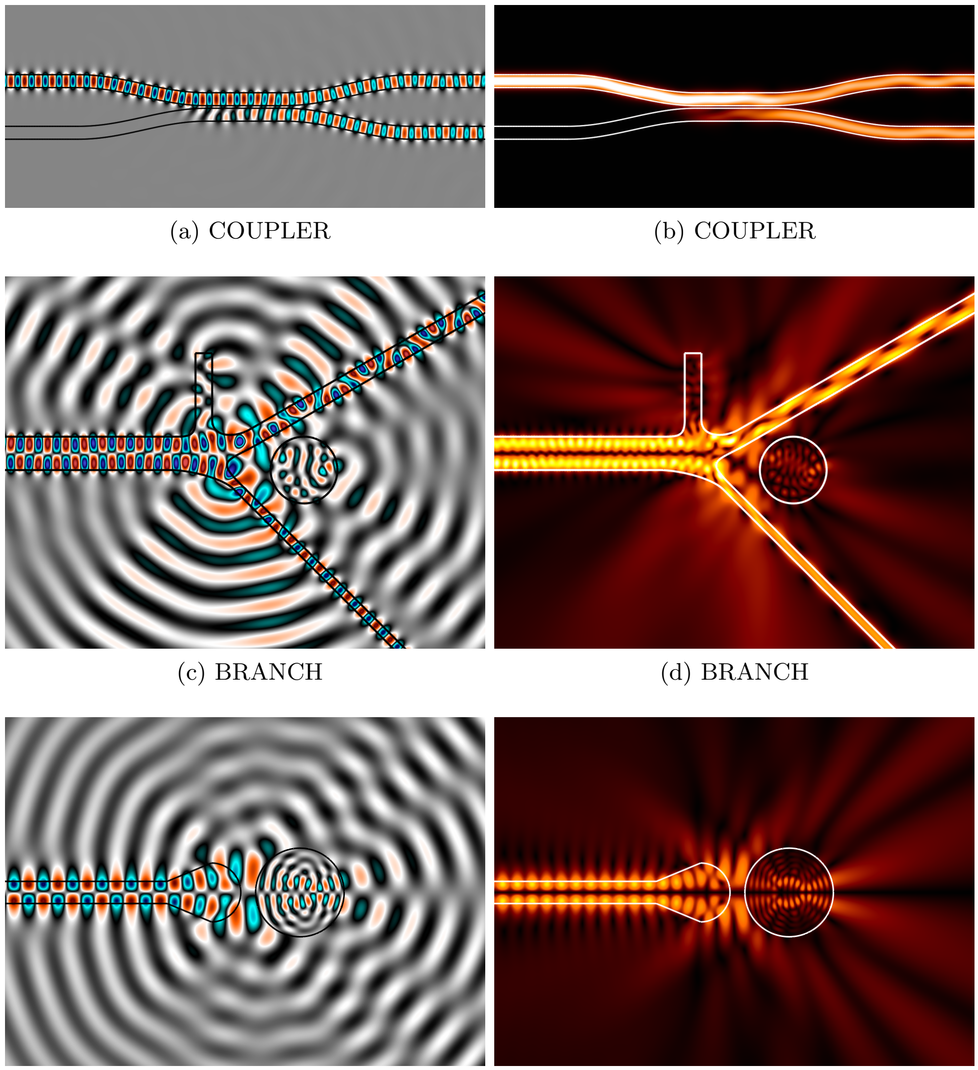
Our failed development of a mode-matching method eventually turned into a fruitful avenue for this problem [21]. Indeed, we realized that the open-waveguide methods (which we created as an element of the sought-for mode-matching method) could be combined with the windowing-function methods discussed in Section 2 to produce high-quality numerical models of dielectric-antenna structures—which are very much in the spirit of the mode-matching method, but which do not suffer from difficulties arising from use of guided modes in small spatial cross-sections. In a nutshell, the new method proceeds as follows: in regions where the geometry departs from perfectly straight waveguides (including possibly launching and termination regions) a full integral equation solution is performed. In the remainder a open-waveguide decomposition is used. The enabling element for such a complex domain decomposition is the WGF method mentioned in Section 2[1].

In summary: this work has lead to Maxwell open-waveguide eigenvalue solvers for waveguides of arbitrary cross-section, as well as a domain decomposition technique based on use of the WGF method, which is demonstrated in Figures 4 and 5. The present implementation assumes two-dimensional spatial models; the three-dimensional solvers are currently under development. We believe this methodology should address the needs originally communicated by Dr. Herscovici, and we would welcome any further interactions with AFRL in these regards.

4 Rectangular integration method

An important new methodology which resulted from this effort concerns a novel *rectangular integration* technique. This method is a substitute for a former "polar-integration" algorithm [16, 24, 25] for integra-

[1] We believe this example reveals the true power of the WGF method: a classical integral-equation approach would likely require solution of integral equations on infinite boundaries which separate the various solution domains. The WGF method does not require any discretizations except for the actual junction/launching/termination regions.



(a) COUPLER (b) COUPLER

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Figure 5: Real part and absolute value of u_w (left and right columns, respectively) produced by the WGF method for several open-waveguide problems and antennas.

tion near Green-function singularities: the new approach delivers essentially the same accuracy levels as the polar-integration approach, but it does not require use of a complex polar integration or otherwise specialized algorithm [13–15, 22, 23] around edges and corners—and it is therefore very well adapted to CAD file descriptions of engineering structures. In view of certain issues concerning evaluation of CAD-file surface attributes, the new approach can be as much as fifty times faster than the previous methodology for scattering surfaces given by standard CAD files.

The rectangular integration method holds great promise towards simulation of mounted-antenna performance, accurate evaluation of radar cross sections, antenna-structure interaction and other challenging electromagnetic scattering and propagation problems. In particular, for example, the rectangular-integration method significantly improves the simulation methodologies for dielectric materials. The multiple improvements observed include high-order resolution of currents around corners and edges and, interestingly, a consequent very significant reduction in the number of GMRES iterations needed for convergence to a given error tolerance. Reductions by factors of ten in the numbers of iterations were observed in many cases. The program in these regards, which is presently highly active, includes simulation, *with high-order accuracy*, of volumetric multimaterial dielectric/PEC structures, as well as modeling of the antenna feeds

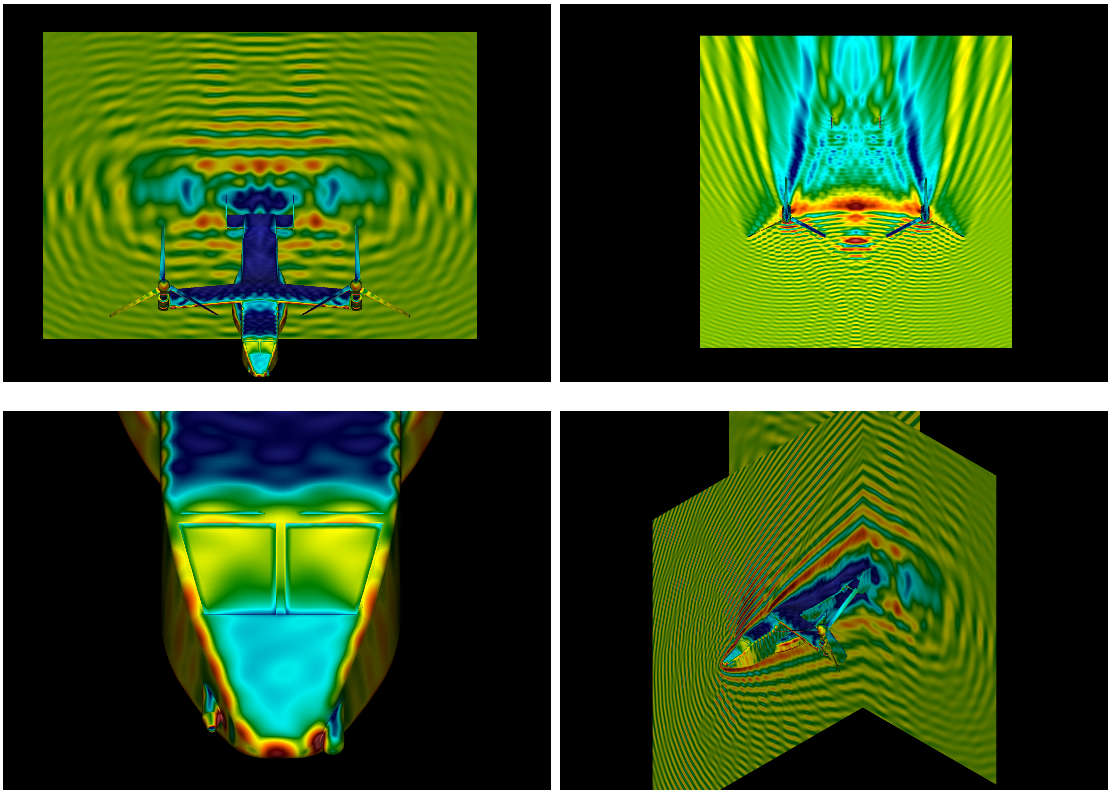

Figure 6: Osprey rotorcraft illuminated by a 900MHz planewave, making it approximately 20λ in electrical length, is used to demonstrate the novel rectangular integration method described in Section 4.

structures, including the transmission line itself. The latter is accomplished by taking advantage of the open-waveguide and closed-waveguide eigenvalue solvers for waveguides of arbitrary cross-section discussed in Section 3 above.

5 Periodic Green function

A number of contributions in the area of rough surfaces and periodic Green functions [9–12] over the span of this contract include use of a new "windowing" approach to greatly accelerate Green-function calculations for rough-surface scattering problems [9], extension of this methodology to periodic arrays of cylinders [10], introduction of acceleration in the periodic solver [11], and extension to three-dimensional periodic problems [12]. The latter problem is very well known and had defied solution since the early twentieth century. The new approach (which is based on use of finite-differences of shifted Green functions for acceleration of convergence even at Wood anomalies) greatly extends the applicability of the Green function methods for periodic scattering problems. The new method has maximum impact in three dimensional problems—for which Green function convergence can be extremely poor, as a result of the existence of large numbers of Wood frequencies. In all, progress in the area of rough surface scattering has been very significant, with applicability to problems in the general areas of metamaterials, oceanic scattering, light-coupling, etc.

As is well known, classical expansions for quasi-periodic Green functions converge extremely slowly, and they of course completely fail to converge at Wood anomalies. A number of methods have been introduced to tackle the slow-convergence difficulty, including the well known Ewald summation method for two and three dimensional problems among many other contributions. Unfortunately, however, none of these methods resolve the difficulties posed by Wood anomalies. Recently, a new quasi-periodic Green function was introduced [9] for the problem of scattering by periodic surfaces which, relying on use of certain linear combinations of shifted free-space Green functions (which amount to discrete finite-differencing of the Green functions) can be used to produce arbitrary (user-prescribed) algebraic convergence order for frequencies throughout the spectrum, including Wood frequencies [27, 29].

A straightforward application of this procedure leads to an operator equation that contains denominators which tend to zero as a Wood anomaly is approached. To remedy this situation a strategy based on use of the Woodbury-Sherman-Morrison formulae is introduced which completely regularizes the problem and provides a limiting solution as Wood frequencies are approached. To our knowledge, this is the first approach ever presented that is applicable to problems of scattering by periodic arrays of bounded obstacles at Wood anomalies on the basis of quasi-periodic Green functions.

6 Explicit and implicit time-domain FC methods of high-order of time accuracy

Important progress occurred over the first year of this contract in the area of FC (Fourier Continuation) methods for Partial Differential Equations in the time-domain [1–8]. Our efforts in these areas have resulted in explicit and implicit solvers for high- and low-frequency problems, for linear and nonlinear equations, and including media such as fluids, solids and vacuum—and combinations thereof. In each of the aforementioned publications a significant milestone was achieved. For example, the contributions [1, 7, 8] provide methods that can be used to enable FC solution of nonlinear equations (such as the Burgers and Navier-Stokes equations) while maintaining high-order accuracy and dispersionlessness with *quasi-unconditional stability*: arbitrarily small values of Δx can be used for a fixed Δt, provided the Δt value adequately samples the problem. In the contribution [2], in turn, methods for FC solution of problems containing variable coefficients were introduced; in particular it was found that certain numerical boundary layers need to be adequately represented in order to ensure accurate solution. Reference [3] introduced an approach that allows for treatment of traction boundary conditions in wave propagation problems in solids [3, 4] (Navier's elastic wave equation) [2]. The thesis [5] uses the Fourier Continuation method in multiple ways: to solve equations, to propagate to distant regions without meshing, etc. The thesis additionally presented an implementation of a three-dimensional FC solver, hybridized with Discontinuous Galerkin, and fully implemented in a GPU computational infrastructure.

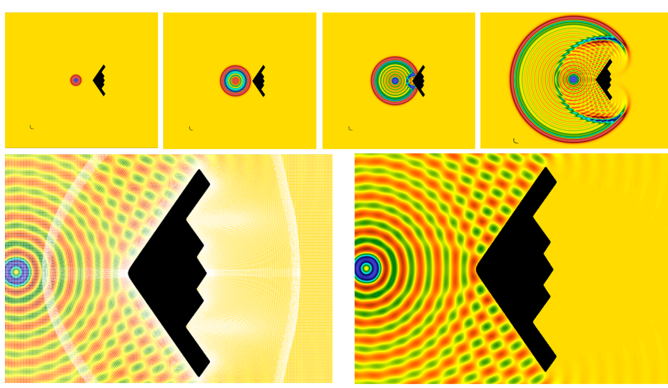

Figure 7: Demonstration of the proposed FC-based time-domain Maxwell evolution algorithm.

In all of these cases the FC method continued to display the excellent qualities observed previously in simpler contexts: high accuracy, exceptionally small dispersion and applicability to completely general configurations. As discussed in the aforementioned contributions, for a given accuracy the FC method

[2] In view of its applications to seismic wave propagation Dr. Amlani's PhD thesis received two awards at Caltech, one in our department and another one for which there is institute-wide competition. The second one is the Demetriades-Tsafka-Kokalis Prize in Engineering & Applied Science for best thesis, publication or discovery in seismo-engineering.

can be anywhere between hundreds and up to millions of times faster, for a given accuracy, than previous alternative solvers.[3] The new methods thus enable solution of previously intractable problems.

7 Conclusions

We believe this work has given rise to significant advances in areas of mathematics and scientific computing closely related to important fields in science and technology. The windowed Green function method provides multiple important contributions, as discussed in Sections 2, 3 and 5. The rectangular integration method presented in Section 4 delivers significant acceleration, up to a factor of fifty, in the *accurate* solution of general scattering problems including structures such as full electrically-large aircraft—hundreds of wavelengths in size. The FC-based solvers mentioned in Section 6, finally, are delivering on their promise of dispersionless solution of general linear and nonlinear partial differential equations in the time domain.

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